C 1

$$\theta_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\varphi_{ij} = c_{ij\alpha\beta}\,\theta_{\alpha\beta} - e_{ij\gamma}\,\mathcal{E}_\gamma$$

$$P_i = e_{\alpha\beta i}\,\theta_{\alpha\beta} + k_{i\alpha}\,\mathcal{E}_\alpha$$

$$\rho\,\ddot{u}_j = F_j = \frac{\partial \varphi_{\alpha j}}{\partial x_\alpha}$$

$$\operatorname{div}\mathcal{E} = -4\pi\operatorname{div}P \qquad \nabla\varphi = -4\pi\operatorname{div}P$$

$$\operatorname{curl}\mathcal{E} = 0 \quad \therefore \mathcal{E} = -\operatorname{grad}\varphi \qquad 3 \qquad \varphi = \int \frac{\operatorname{div}P}{r}\,d\tau$$

$$\rho\,\ddot{u}_j = c_{\delta j\alpha\beta}\frac{\partial \theta_{\alpha\beta}}{\partial x_\delta} - e_{\delta j\gamma}\frac{\partial \mathcal{E}_\gamma}{\partial x_\delta}$$

$$\rho\,\ddot{u}_j = \frac{1}{2}c_{\delta j\alpha\beta}\frac{\partial}{\partial x_\delta}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right) - \;''$$

$$= c_{\delta j\alpha\beta}\frac{\partial^2 u_\alpha}{\partial x_\delta\,\partial x_\beta} - e_{\delta j\gamma}\frac{\partial \mathcal{E}_\gamma}{\partial x_\delta}$$

In general this case permits of no wave eqn. in the strict sense rather a wave integro-diff eqn. In the case of plane waves we may proceed as follows:

Let n be a variable measured normal to the phase planes and let n_i be the unit normal. If Q is any quantity

$$Q = f(n\,t)$$

$$n_i\frac{\partial Q}{\partial n} = \frac{\partial Q}{\partial x_i} \quad \text{or} \quad \nabla Q = \frac{\partial Q}{\partial n}\,\hat{n}$$

$$\frac{\partial P_i}{\partial x_i} = e_{\alpha\beta i}\frac{\partial \theta_{\alpha\beta}}{\partial x_i} + k_{i\alpha}\frac{\partial \mathcal{E}_\alpha}{\partial x_i}$$

$$-4\pi\frac{\partial P_i}{\partial x_i} = \frac{\partial \mathcal{E}_i}{\partial x_i} \quad \therefore \quad K_{i\alpha}\frac{\partial \mathcal{E}_\alpha}{\partial x_i} = -4\pi\,e_{\alpha\beta i}\frac{\partial \theta_{\alpha\beta}}{\partial x_i}$$

in which $\quad K_{i\alpha} = \delta_{i\alpha} + 4\pi k_{i\alpha}$

$$\frac{\partial Q}{\partial x_i} = n_i \frac{\partial Q}{\partial n} \qquad \mathcal{E}_i = n_i \mathcal{E}_n$$

$$\frac{\partial \mathcal{E}_\alpha}{\partial x_\beta} = n_\alpha n_\beta \frac{\partial \mathcal{E}}{\partial n}$$

$$\rho \ddot{u}_j = c_{\delta j \alpha \beta} \frac{\partial^2 u_\alpha}{\partial n^2} n_\delta n_\beta - e_{\delta j \gamma} \frac{\partial \mathcal{E}_n}{\partial n} n_\delta n_\gamma$$

$$K_{i\alpha} n_\alpha n_i \frac{\partial \mathcal{E}_n}{\partial n} = -4\pi\, e_{\alpha\beta i} \frac{\partial^2 u_\alpha}{\partial x_i \partial x_\beta}$$

$$= -4\pi\, e_{\alpha\beta i} n_i n_\beta \frac{\partial^2 u_\alpha}{\partial n^2}$$

So
$$\rho \ddot{u}_j = \left[c_{\delta j \alpha\beta} n_\delta n_\beta + 4\pi \frac{e_{\delta j \gamma} n_\delta n_\gamma e_{\alpha\beta i} n_i n_\beta}{K_{i\alpha} n_\alpha n_i} \right] \frac{\partial^2 u_\alpha}{\partial n^2}$$

$$= \left[c_{\delta j \alpha\beta} n_\delta n_\beta + 4\pi \frac{e_{\delta j \gamma} n_\delta n_\gamma e_{\alpha\beta i} n_i n_\beta}{K_{i\alpha} n_\alpha n_i} \right] \frac{\partial^2 u_\alpha}{\partial n^2}$$

$$+4\pi \frac{e_{j11}\, e_{\alpha 11}}{K_{11}}$$